26573
new 79-01

COMPLIMENTARY COPY
Please Credit Ontario Hydro Research if
Reproduced in Whole or in Part.

ontario hydro
research division

GENERATION OF SYNTHETIC FRACTURE PARAMETERS FOR CRACK NETWORK ANALYSIS

Report No 82-173-K

A.H.C. Chan, L.D. Wilson, A.L. Punhani
Operations Research Department

ABSTRACT

This report outlines a basic approach to generating synthetic information on the fracture structure in the rock surrounding a nuclear waste repository. The approach is to analyze geological field data; construct a statistical model of fractures; use a Monte Carlo sampling technique to generate sets of fracture descriptions; find networks of intersecting fractures; and transfer a description of these networks to WNRE's model of groundwater flow through discrete fractures (FLOWNET). This report concentrates on the statistical model of fractures and a trial application of this model. The need for decisions in three areas is identified.

Work done for
Atomic Energy of Canada Limited
Whiteshell Nuclear Research Establishment
under the Techanical Assistance Program

job	file	date	report no.
740672-102	809.730	December 28, 1982	82-173-K

TABLE OF CONTENTS

		Page
1.	INTRODUCTION	1
	1.1 Model of a Fracture	1
	1.2 The Need for a Statistical Technique	2
	1.3 Generating the Fracture Structure	3
2.	GENERATION OF SYNTHETIC FRACTURES	3
	2.1 Preliminary Data	4
	2.2 Assumptions about Missing Data	7
	2.3 The Simulation Procedure	10
	2.4 Discussion	16
3.	DETERMINATION OF FRACTURE NETWORKS	17
4.	SUMMARY AND RECOMMENDATIONS	17
REFERENCES		19
APPENDIX I	Spacing of Simulated Fractures	20
APPENDIX II	Verification of the Simulation Model	22
APPENDIX III	Trial Application of the Fracture Generation Simulation	26
APPENDIX IV	Modification to Accommodate the Observed Lineaments	30
DISTRIBUTION		last page

ontario hydro
research division

To Mr. F.J. Kee
Director of Research

GENERATION OF SYNTHETIC FRACTURE PARAMETERS FOR CRACK NETWORK ANALYSIS

1. INTRODUCTION

An important part of the pathway analysis study in Canada's Nuclear Waste Management Program is the development of methods for estimating the flow of groundwater through fractures in rock surrounding a nuclear waste repository. This report outlines an approach based on modelling flow through discrete fractures. An important part of the pathway analysis study in Canada's Nuclear Waste Management Program is the development of methods for estimating the flow of groundwater in rock surrounding a nuclear waste repository. The Whiteshell Nuclear Research Establishment (WNRE) is developing a discrete fracture flow code (FLOWNET) to calculate the flow of groundwater through fractures, and has asked Ontario Hydro's Operations Research Department to develop a statistically based technique for generating information that describes the fracture structure (ie, the complete specification of the locations, orientations, dimensions and intersections of significant fractures).

Since the majority of fractures in a rock mass cannot be detected without exhaustive borehole exploration, it is necessary to use statistical inference methods to extrapolate the characteristics of observed surface and subsurface features to estimate the complete fracture structure. It is anticipated that Monte Carlo sampling from probability distributions of fracture parameters will be repeatedly used to construct plausible fracture structures for groundwater flow analyses. The results will be used to construct probability distributions of groundwater flow factors of interest in pathway analysis.

The purpose of this report is to outline the basic approach being taken to generate synthetic fracture structure information for further analysis by WNRE's FLOWNET program. The concept is to represent a fracture by 8 parameters. The statistical procedure, then, is to select samples of 8-tuples of these parameters from appropriate probability distributions to represent the fractures in the rock mass. The actual geometric exercise and computer code to calculate and code the coordinates of the fractures and their intersections is described in Reference 3.

1.1 Model of a Fracture

The mathematical model for this analysis of fracture structures assumes that each fracture is shaped as a rectangular prism and so can be completely specified by a location reference point, P(x,y,z), and the parameters θ, ω, e, d and a, as shown in Figure 1. The parameters are defined as follows:

e is the strike length, d is the dip length and a is the aperture of the fracture. Strike angle, θ, is the angle that the strike direction makes with the positive y-axis and the dip angle, ω, is the angle measured clockwise from the surface to the plane of the fracture with reference to the strike direction. Location reference point, P(x,y,z), is the western-most point on the fracture (for a fracture whose strike direction is north-south, the location reference point is the southern-most point on the fracture). A fracture is represented by an 8-tuple:

$$(x, y, z, \theta, \omega, e, d, a).$$

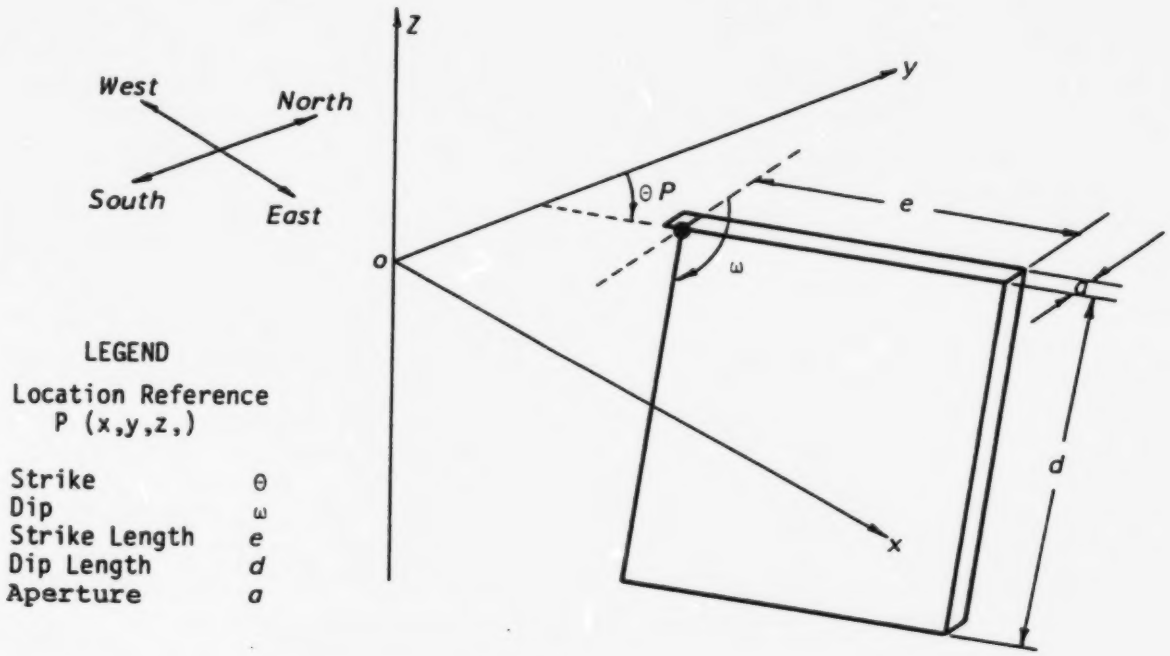


FIGURE 1

1.2 The Need for a Statistical Technique

If all the parameters of each fracture in a rock mass were known, then its fracture structure could be determined by an exercise in three dimensional geometry.

Unfortunately, it is not possible to locate and measure most fractures in a rock mass without destroying its usefulness as a site for a nuclear waste repository. Therefore, it is necessary to use the limited amount of field data available from surface observations and borehole investigations to estimate the joint probability distribution of fracture parameters. Then it must

be assumed that the actual fracture structure of the rock mass is one of the infinite number of possible structures that are consistent with the field data. Such a structure can be generated mathematically by drawing random samples of fracture parameters from the estimated joint probability distribution.

This approach depends heavily on the availability of geological field data that has been collected and stored in a manner that allows for a statistical analysis of the variability and interrelationships of the location reference point coordinates and parameters of the fractures. For example, it is important to learn if there are strong correlations between parameters such as ω and θ, and a, θ and e. If there are strong correlations, then the data should be used to estimate joint probability distribution functions for the parameters. If correlations are weak, then a probability distribution function should be estimated for each independent parameter.

1.3 Generating the Fracture Structure

Conceptually, if the probability distribution function that defines the variability and interrelationships of the location reference point coordinates and parameters of the fractures is known (or can be estimated with acceptable accuracy), then a Monte Carlo simulation method can be used to create sets of location reference points and the corresponding parameters of the fractures to represent all the fractures in the rock mass under study. These fractures can be examined to identify the ones that form network structures through which groundwater may flow. A description of these networks (the location, size and orientation of the fractures and their intersections) can then be transferred to WNRE's discrete fracture flow computer code to calculate groundwater movement through the rock mass.

Ideally, this process should be repeated a large number of times in order to establish reliable estimates of the range of groundwater flow patterns that may be possible.

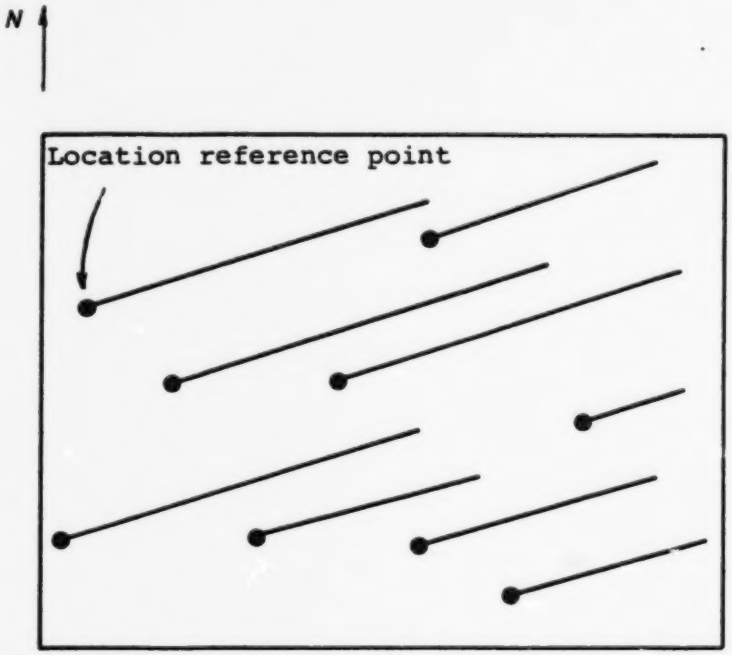
2. GENERATION OF SYNTHETIC FRACTURES

The purpose of the simulation is to produce an appropriate number of fractures (x, y, z, θ, ω, e, d, a) by drawing random samples from appropriate probability distributions of the field measured parameters λ, Q, θ, ω, e, d and a. These parameters are defined in Figure 1 except for λ and Q (λ is the horizontal spacing of fractures and Q is the vertical separation of fractures). The parameters λ and Q will be used to generate location reference parameters x, y, z (unmeasurable in the field) as described in the next section. If the probability distributions have been carefully developed from an analysis of field geological observations, then the set of simulated fractures will tend to resemble the pattern of fractures found in the real rock mass. It may be necessary to divide the rock mass under study into a number of relatively homogenous zones to preserve its structural characteristics in the simulation.

2.1 The Basic Approach

The most important part of the procedure for generating synthetic fractures is to use the information of fracture spacings (ie, λ and Q) to create the location reference points of fractures. The generated fractures should have the same spacing distribution as observed in the field.

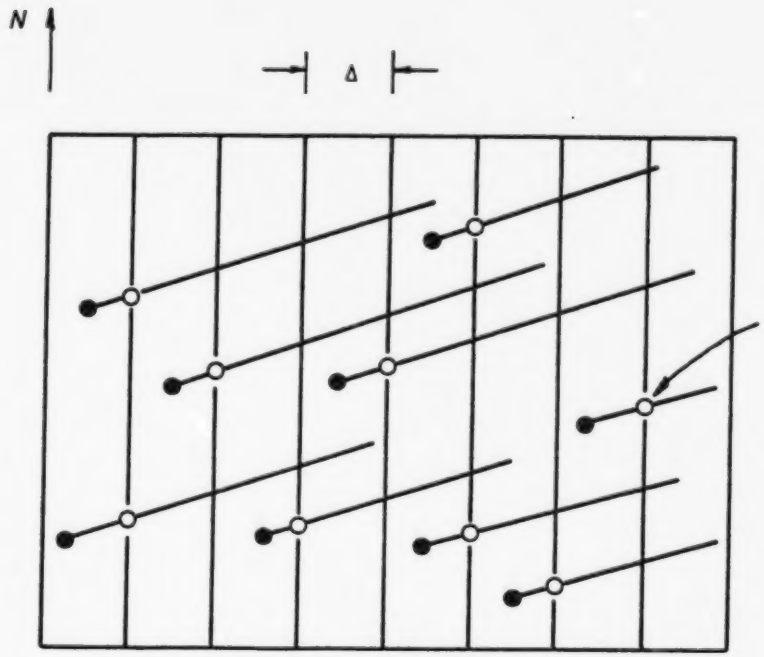
To illustrate the generation of surface fracture location reference points by the parameter λ, consider Figure 2.



Fractures of a Joint Set

FIGURE 2

Let the rectangle be the surface of the rock mass, the straight lines be the fractures on the surface and the black dots be the location reference points. If a set of equally spaced vertical lines (north-south) is superimposed on the rectangle (Figure 3), then these vertical lines will intersect the fractures at one or more locations. Let the white dots represent the points of intersection of the fractures and the lines which are closest to the black dots. As the size of the spacing, Δ, becomes small (close to zero), the white dots approach the black dots. Hence, if Δ is sufficiently small, all location reference points will lie on the vertical lines. The spacing of the reference points on the vertical lines can be deduced from the spacing of the fractures as described in Appendix II.



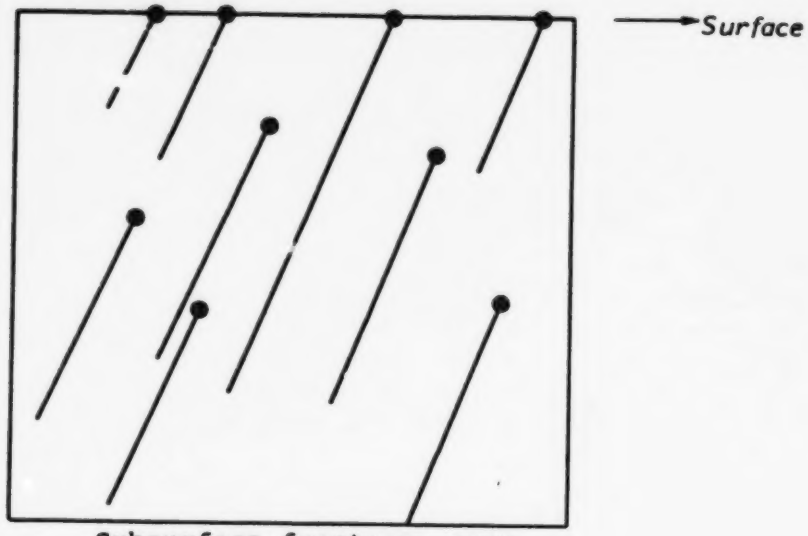
Intersection of Vertical Lines and Fractures

FIGURE 3

To illustrate the generation of the location reference points of the subsurface fractures (fractures that do not extend to the surface) by the parameter Q, consider Figure 4. Let the rectangle represent a vertical cross section of the rock mass and the lines represent the fractures intersecting this vertical plane.

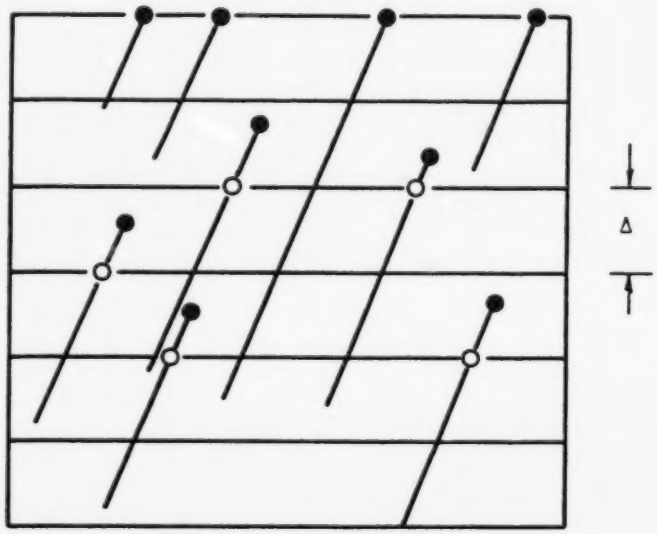
Again, if a family of equally spaced horizontal lines is superimposed on the rectangle where each line represents a plane parallel to the surface, then fractures will intersect the horizontal lines, indicated by the white dots in Figure 5.

Similarly, if the spacing of the lines, Δ, is sufficiently small, it can be assumed that each fracture location reference point lies on one of these horizontal planes (as explained previously). If the vertical separation of fractures, Q, (eg, observed from boreholes) is given, it can be converted to the spacing of fractures on each horizontal surface as described in Appendix I. The method of Appendix II can be used to generate the fracture location reference point on each of these surfaces.



Subsurface fractures seen
from a vertical section

FIGURE 4

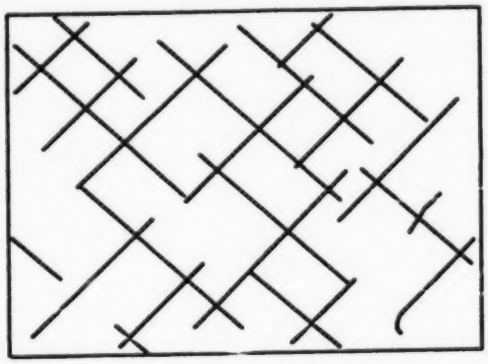


Intersection of fractures and horizontal
planes

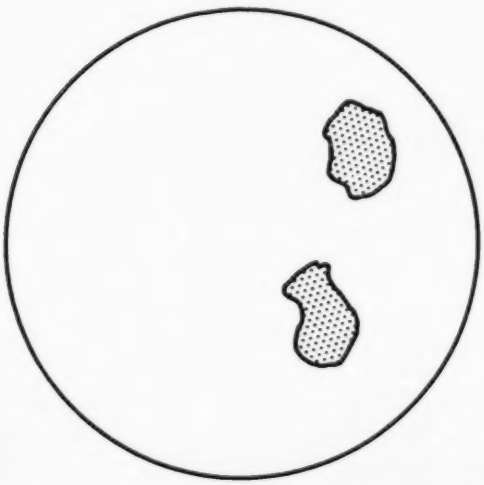
FIGURE 5

2.2 Data

In order to develop the fracture network model, plausible data on fractured hard rock were required. At the time this report was prepared, the results of Dr. K.D. Raven's work at Chalk River were available, so they were used. Dr. Raven's report/1/ presents distributions of fracture orientations (strike and dip) in the form of stereoplots (see also Pit Slope Manual/2/). These plots allow the fractures to be divided subjectively into different joint sets (see Figure 6). The joint sets could be horizontal or non-horizontal. A horizontal joint set is the family of fractures approximately parallel to the surface.



Fractures of two joint sets



Stereoplot of the orientations of the two joint sets

FIGURE 6

The procedure for analyzing geological field data is described in the following three steps:

(a) Let S_i, $i = 1,2,..,I$ be the I non-horizontal joint sets. Assume that, within set S_i, the parameters, θ_i, ω_i, e_i, d_i, a_i and λ_i, follow probability distributions Θ_i, Ω_i, E_i, D_i, A_i and Λ_i respectively. Their mean values are given by $\bar{\theta}_i$, $\bar{\omega}_i$, $\bar{e}_i$, $\bar{d}_i$, $\bar{a}_i$ and $\bar{\lambda}_i$.

Let the horizontal joint set be denoted by the symbol S_h. For S_h, let $\theta_h = \pi$ and $\omega_h = 0$ for each fracture; also, let e_h and a_h follow the distribution E_h and A_h with means $\bar{e}_h$ and $\bar{a}_h$.

For mathematical reasons when $\omega_i = \pi/2$, S_i should be split into two clusters, one having $\omega_i \geq \pi/2$ and the other having $\omega_i < \pi/2$. Then the mean dip angles of these two elements will be different from $\pi/2$.

Similarly if $\theta_i = 0$, divide S_i into two clusters, one having $\theta_i > 0$ and the other having $\theta_i < 0$. For clusters of $\theta_i < 0$, put $\theta_i = \pi - \theta_i$.

(b) The dip length of fractures, d, is assumed to be defined by $d = \rho e$ (where e is the strike length and ρ is a constant); for horizontal fractures $\rho = 1$.

(c) The vertical separation of fractures in a joint set is determined from the borehole data. Let $Q^*(z)$ be the expected number of fractures in the set S_i that intersect a line segment between (x, y, 0) and (x, y, z), for any (x, y). This function depends on the vertical separation of sub-surface fractures and is needed in the simulation to generate sub-surface fractures as explained in Section 2.1.

The number of sub-surface fractures intersecting this line segment is:

$$Q_i(z) = Q_i^*(z) - \frac{\bar{d}_i \left|\cos\bar{\omega}_i\right|}{\bar{\lambda}_i}$$

$$Q_h(z) = Q_h^*(z)$$

where $\dfrac{\bar{d}_i \left|\cos\bar{\omega}_i\right|}{\bar{\lambda}_i}$ is the number of surface fractures intersecting this line (for proof see proposition II in Appendix II).

2.3 The Simulation Procedure

To simplify the description, no observed fractures are considered here. In the following section, the modification of this procedure to accommodate the observed fractures is described. The simulation procedure can be outlined in a general form. Consider a body of rock of size $D_1 \times D_2 \times D_3$ as shown in Figure 7.

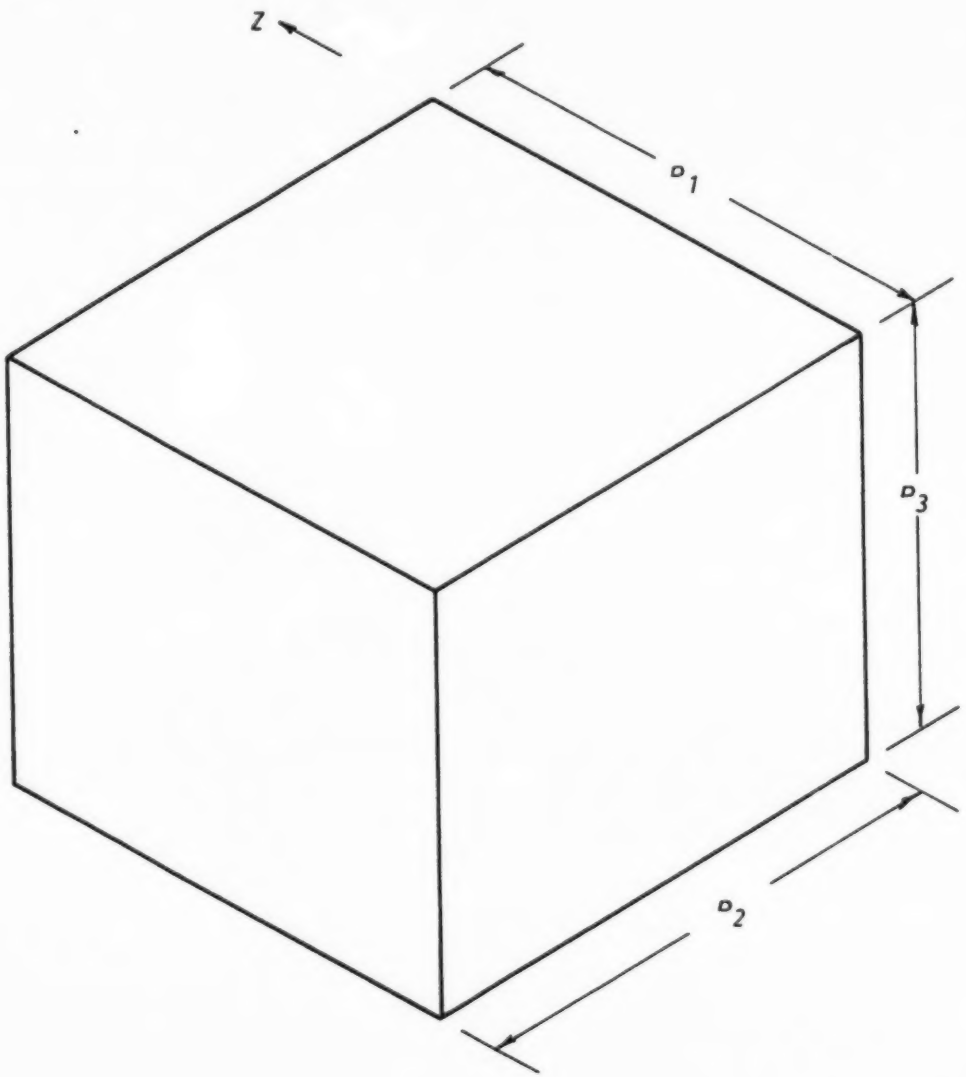


FIGURE 7

The basic idea of the simulation procedure is:

1. Generate a number of points within the block $D_1 \times D_2 \times D_3$ representing the location reference points of the simulated fractures.

2. Draw a random sample of the parameters θ_i, ω_i, e_i and a_i for these fractures using probability distributions Θ_i, Ω_i, E_i and A_i, respectively.

3. Generate the horizontal fractures using $Q_h(z)$ and E_h.

The following four steps describe the simulation procedure in details:

STEP 1: SAMPLING THE LOCATION REFERENCE POINTS

For Surface Fractures

As described in Section 2.1, it can be assumed that fracture location reference points lie on the north-south equally spaced lines (ie, parallel to y-axis, spaced Δ apart). Hence, a set of equally spaced lines is superimposed on the $D_1 \times D_2$ surface perpendicular to the x-axis. These lines are spaced with a separation Δ, where Δ is chosen so that

$$\Delta \leq \min_{1 \leq i \leq I} \{\bar{e}_i \sin\bar{\theta}_i, \bar{d}_i \sin \bar{\omega}_i\},$$

and D_1, D_2, D_3 are divisible by Δ (theoretically, Δ has to be sufficiently small). These lines will pass through the points $j\Delta$, $j = 0, 1, 2, \ldots, \frac{D_1}{\Delta}$ on the x-axis (see Figure 8).

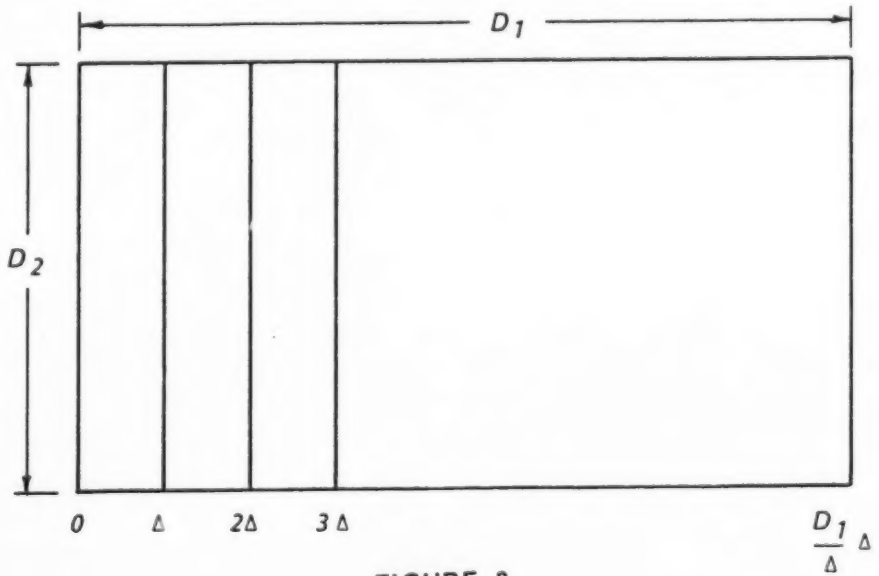


FIGURE 8

After these lines have been generated, the next step is to locate points on each line that will represent location ref r-ence points (x,y,z) for fractures. This must be done to preserve the observed fracture spacing for each joint set. On the surface plane (z=0), the mean spacing of points on each line will be: $\bar{\lambda}_i \bar{e}_i/\Delta$.

If fractures of spacing $\lambda_i e_i/\Delta$ are generated from the line at x = 0, the expected spacing between the lines at x = 0 and $x = [\bar{e}_i \sin\bar{\theta}_i/\Delta] \cdot \Delta$[1] will be greater than $\bar{\lambda}_i$ because no fracture is overlapping at the line x = 0. To compensate this, fractures should be generated from the line x = -m, where

$$m = \max \left\{ \left[\frac{\bar{e}_i \sin\bar{\theta}_i}{\Delta} \right] + 1,\ i = 1, 2, 3, \ldots, I \right\} .$$

For the same reason, if the fractures are restricted to y between 0 and D_2, the expected spacing between y = 0 and $y = \bar{e}_i \cos\bar{\theta}_i$ if $\theta_i \leq \pi/2$ or between $y = D_2 + \bar{e}_i \cos\bar{\theta}_i$ and $y = D_2$ when $\theta_i > \pi/2$ will be greater than λ_i. Therefore, fractures should be generated between $y = \min \{0, -\bar{e}_i \cos\bar{\theta}_i\}$ and $y = \max \{D_2, D_2 - \bar{e}_i \cos\bar{\theta}_i\}$.

Since it is assumed that the location reference points lie on the vertical lines, then x-coordinates are multiples of Δ, ie, $j\Delta$, j = -m, ..., 0, 1, 2 ..., D_1/Δ and the z-coordinate is z = 0. It still remains to generate the y-coordinates. Suppose that we are generating fractures on the line at $x = j\Delta$. Then, the spacing of the fractures on this line follows the distribution $\Lambda_i'(u) = \Lambda_i(u\Delta/e_i)$ (see Appendix II). Select a se-quency of random numbers $S_{i0}(j,r)$, r = 1, 2, 3, 4 ... from the distribution Λ_i'. Then, the y coordinate of the first location reference point is $S_{i0}(j,1) + \min \{0, -\bar{e}_i \cos\bar{\theta}_i\}$. In general, the y coordinate of the rth location reference point is (see Figure 9)

$$y = s_{i0}(j,1) + s_{i0}(j,2) + \ldots + s_{i0}(j,r) + \min \{0, \bar{e}_i \cos\bar{\theta}_i\}$$

The generation of location reference points on this line will terminate at $n_{i0}(j)$, where

---

[1] $\left[\frac{\bar{e}_i \sin\bar{\theta}_i}{\Delta} \right]$ is the largest integer less than or equal to $\frac{\bar{e}_i \sin\bar{\theta}_i}{\Delta}$.

$$\sum_{r=1}^{n_{i0}(j)} s_{i0}(j,r) \leq D_2 + \left| \bar{e}_i \cos\bar{\theta}_i \right| \leq \sum_{r=1}^{n_{i0}(j)+1} s_{i0}(j,r) \qquad (2.1)$$

for every $i = -m, \ldots, 0, 1, \ldots, D_1/\Delta$ (see Figure 9).

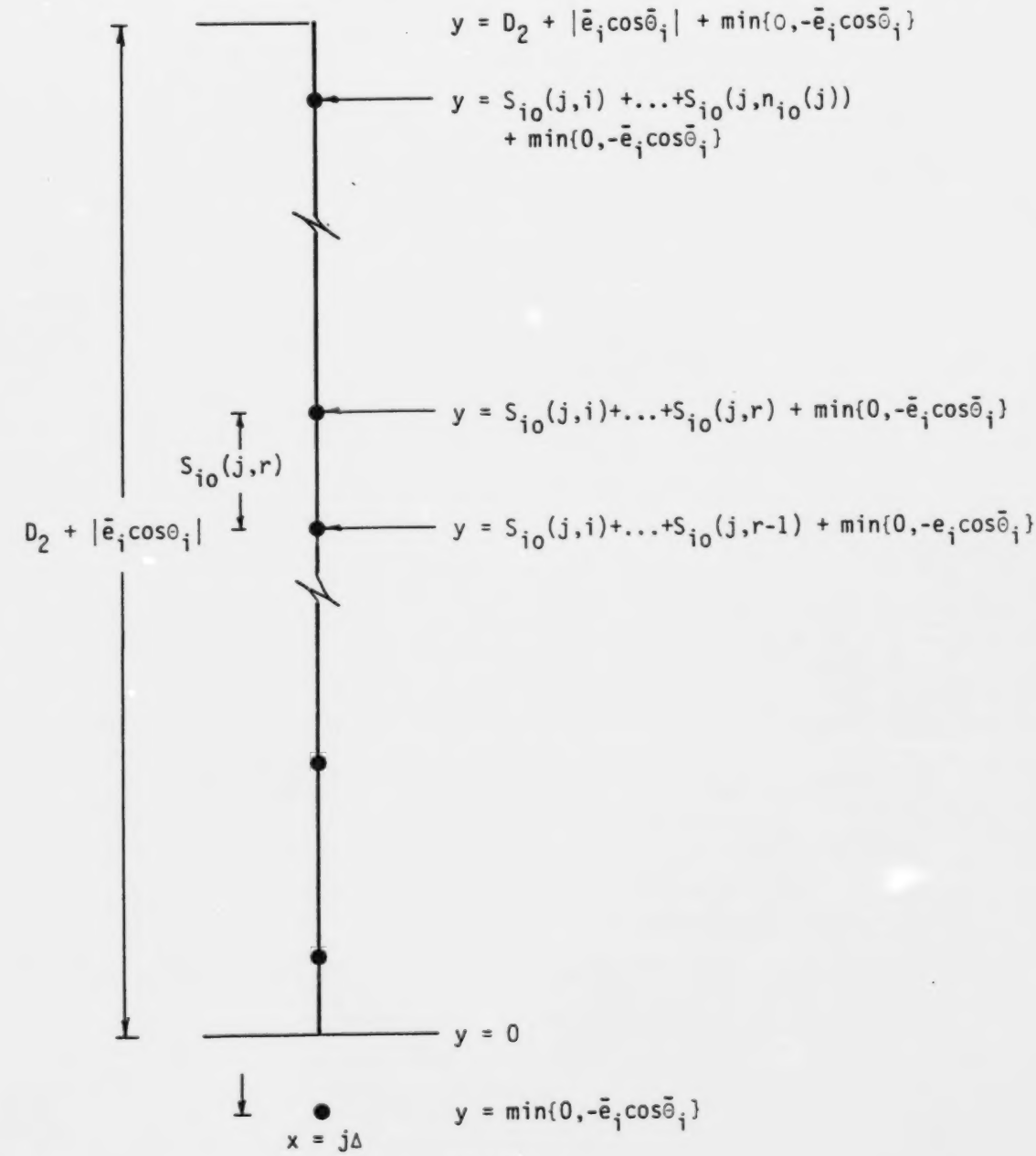

y = D2 + |ēi cosθ̄i| + min{0,-ēi cosθ̄i}
y = Sio(j,i) +...+Sio(j,nio(j)) + min{0,-ēi cosθ̄i}
y = Sio(j,i)+...+Sio(j,r) + min{0,-ēi cosθ̄i}
Sio(j,r)
D2 + |ēi cosθi|
y = Sio(j,i)+...+Sio(j,r-1) + min{0,-ei cosθ̄i}
y = 0
y = min{0,-ēi cosθ̄i}
x = jΔ

FIGURE 9

Then, the location reference points are:

$$P(x,y,z) = P\left[j\Delta, \sum_{r \le R} s_{i0}(j,r) + \min\{0, -\bar{e}_i \cos\bar{\theta}_i\}, 0\right]$$

where $j = -m, \ldots, -1, 0, 1, 2, \ldots, \frac{D_1}{\Delta}$

and $R = 1, 2, \ldots, n_{i0}(j)$.

For the Subsurface Fractures

As explained in Section 2.1, subsurface fractures are assumed to start at one of the horizontal planes $z = k\Delta$, $k = 1,2,\ldots,D_3/\Delta$. On each plane $z = -k\Delta$, a set of vertical (parallel to the y-axis) equally spaced lines with spacing Δ is superimposed. It is also assumed that the location reference points lie on these lines (Figure 10).

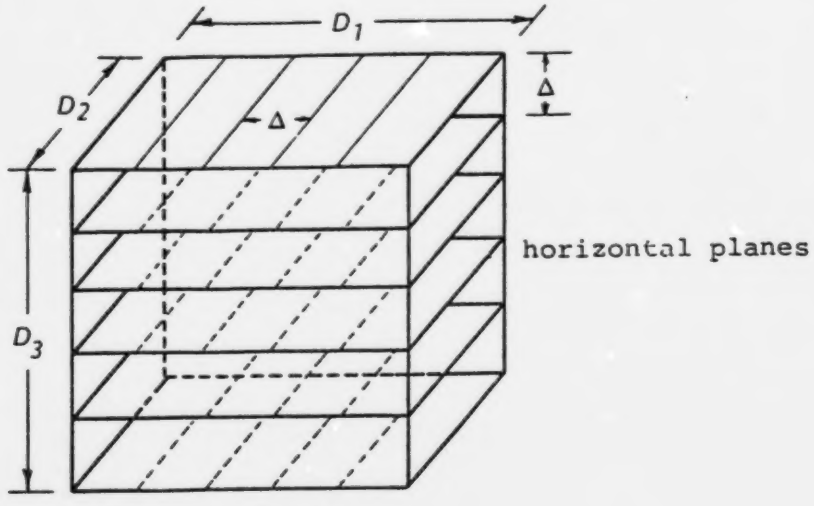


FIGURE 10

As explained in Appendix I, in order to preserve the fracture spacing that can be observed by inspecting boreholes, the mean spacing of points on sub-surface projections of the lines in Figure 8 is:

$$\bar{s}_{ik} = \frac{\bar{d}_i \, \bar{e}_i \left|\cos\bar{\omega}_i\right|}{\Delta\{Q_i[(k+1)\Delta] - Q_i(k\Delta)\}}$$

for the joint sets with $\bar{\omega}_i \neq \frac{\pi}{2}$, and $k = 1, 2, \ldots, \frac{D_3}{\Delta}$.

Assuming that fracture spacing on the surface planes at $z_k = k\Delta$, $k = 1, 2, \ldots, D_3/\Delta$, follows a distribution Λ_i' with mean $\bar{s}_{ik}$. This distribution can be estimated by the borehole information.

Similar to the surface fractures, a sequence of random number $s_{ik}(j,r)$, $r = 1, 2, \ldots, n_{ik}(j)$ is sampled where

$$\sum_{r=1}^{n_{ik}(j)} s_{ik}(j,r) \leq D_2 + \left|\bar{e}_i \cos\bar{\theta}_i\right| \leq \sum_{r=1}^{n_{ik}(j)+1} s_{ik}(j,r),$$

and, $j = -m\ldots, -1, 0, 1, 2, \ldots, \frac{D_1}{\Delta}$.

Then, assign the location reference point as

$$P(x,y,z) = P(j\Delta, \sum_{r \leq R} s_{ik}(j,r) + \min\{0, -\bar{e}_i \cos\bar{\theta}_i\}, z_k)$$

where $j = -m, \ldots, -1, 0, 1, 2, \ldots, \frac{D_1}{\Delta}$,

$k = 1, 2, \ldots, \frac{D_3}{\Delta}$, and

$R = 1, 2, \ldots, n_{ik}(j)$.

STEP 2: SELECTING THE REST OF THE PARAMETERS

After generating the location reference points, a set of parameters, namely θ, ω, e, d and a, corresponding to each of the reference points is required to complete a synthetic fracture. Ideally, if the joint probability distribution of the eight parameters (x, y, z, θ, ω, e, d, a) is known, say F(x, y, z, θ, ω, e, d, a), then this set of parameters can be sampled from the conditional distribution $F(\theta, \omega, e, d, a \mid x, y, z)$ for each generated reference point (x, y, z). Since the joint probability distribution is not known, and θ, ω, e and a are assumed to be independent with $d = \rho e$, sample θ, ω, e and a from distributions Θ_i, Ω_i, E_i and A_i for each location reference point (x, y, z) to complete a set of synthetic fractures.

STEP 3: GENERATING THE REMAINING NON-HORIZONTAL FRACTURE SETS

Increase i by 1 and repeat steps 1 and 2 for the S_i fractures up to i = I.

STEP 4: GENERATING THE HORIZONTAL FRACTURES

The information on horizontal fractures available might be the distributions A_h, E_h and Q_h as defined in Section 2.2(b).

These distributions cannot be converted to the distributions of location reference points as proposed for non-horizontal fractures. Therefore, it is assumed that the location reference points of the horizontal fractures are equally likely to occur at any point (x, y) beneath the surface. On the other hand, the expected number of horizontal fractures intersecting any vertical (to the surface) line between 0 and -z is $Q_h(z)$.

Using the same reasoning as in Step 1 for the other joint sets, the selection of reference points should start at $x = -\bar{e}_h$ and $y = -\bar{e}_h$ (see Figure 11; $\bar{e}_h$ is the expected value of E_h).

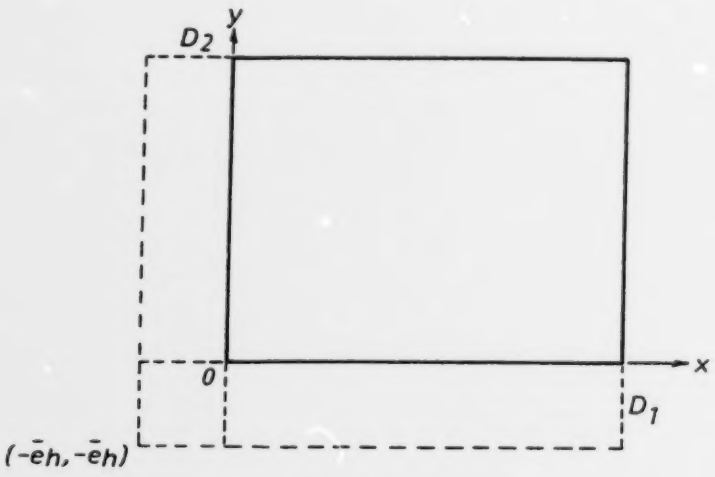


FIGURE 11

Define $B = Q_h(D_3)$ and $Q_h'(z) = \frac{1}{B} Q_h(z)$. The location reference points are sampled from the distribution

$$U(-\bar{e}_h, D_1) \cdot U(-\bar{e}_h, D_2) \cdot Q_h'$$

where U represents a uniform distribution.

For each reference point (x_n, y_n, z_n), sample e_n from E_h and a_n from A_h.

Let

$$\epsilon_n = (\min\{e_n+x_n, D_1\} - \max\{0, x_n\})\ (\min\{e_n+y_n, D_2\} - \max\{0, y_n\})$$

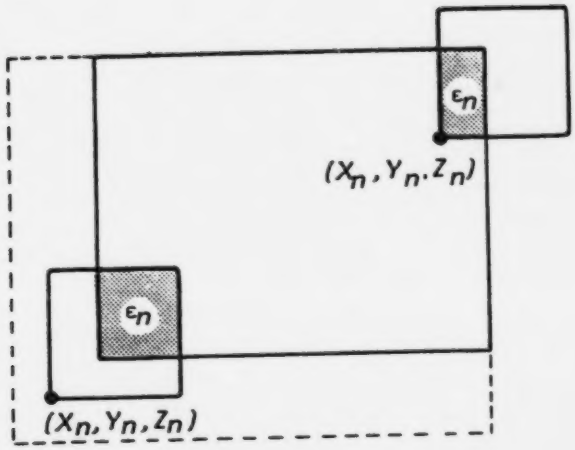


FIGURE 12

As illustrated in Figure 12, ε_n is the area of the shaded region. The selection of samples will terminate at n_h when

$$\sum_{n=1}^{n_h} \varepsilon_n \leq D_1 D_2 B \leq \sum_{n=1}^{n_h+1} \varepsilon_n$$

By putting $d_n = e_n$, n_h horizontal fractures have been generated, namely,

$(x_n, y_n, z_n, 0, \pi, e_n, d_n, A_n)$, $n = 1, 2, \ldots, n_h$.

It may be noted that the expected number of fractures intersecting any vertical line between 0 and -z is $Q_h(z)$.

2.4 DISCUSSION

In the above section, a procedure to generate a completely synthetic fracture structure was developed in detail. In some situations fractures such as major lineaments are observable in the area. In such cases, the generating procedure should be modified accordingly so that the major lineaments are included as part of the synthetic fracture structure. Appendix IV gives the details of this modification.

Appendix III outlines and discusses the assumptions made for a trial application of the fracture simulation model, undertaken to produce test data for WNRE's discrete fracture flow computer

code. Since this was an exercise to gain experience with the model and the computer code, no attempt was made to achieve a high degree of realism. The computer code utilizing this statistical approach is described in /3/. The output information of this computer code is used by WNRE's FLOWNET program.

3. DETERMINATION OF FRACTURE NETWORKS

Techniques for the identification of fracture intersections and numbering systems for tagging edges and surfaces of intersecting fractures have been developed as part of this project. The mathematical and computational aspects of this work are described in /3/.

4. SUMMARY AND RECOMMENDATIONS

This report outlines the basic approach being taken to generate synthetic fracture structure information for further analysis by WNRE's model of groundwater flow through a network of discrete fractures. The modification to accommodate existing fractures such as major lineaments is also explained.

Since detailed information on the structure of fracture networks in rock bodies is impossible to obtain, it is proposed that a statistical description of geological field data be used to create a model of these networks. Monte Carlo sampling from this model will be used to generate a large number of sets of synthetic fracture structure information (sets of fracture parameters that are consistent with field data reported in /1/) for groundwater flow analysis. A computer code is being developed to generate sets of fracture descriptions and find networks of intersecting fractures to be used by WNRE's FLOWNET code.

This model is flexible enough to accommodate any adjustments needed when additional information becomes available.

RECOMMENDATIONS

(a) Plan geological field studies to collect data in a form needed for this analysis.

In particular, effort must be made to study distributions of the parameters and their relationships. For example, the following questions need to be answered:

1. Does the orientation depend on the location of the fractures?

2. Does the spacing vary from location to location?

3. Do fracture densities and apertures vary with depth?

4. What are the distributions of the strike length and dip length?

(b) Set the objectives of the groundwater flow analyses in specific terms.

In other words, what is the ultimate goal of this analysis? Is it the exit time of radionuclides from the repository? If the objectives are clearly defined, this will help to formulate the Monte Carlo simulation method accordingly.

(c) Develop a plan for finding the most cost-effective way to carry out the groundwater flow analyses.

A trial run of the computer code indicates that the cost is extremely high. The feasibility of the whole program may depend on the cost-effectiveness of the analysis.

Approved:

J.G. Cassan
Manager
Operations Research Dept

Submitted:

A.H.C. Chan
Mathematician
Reliability and Statistics Section

Reviewed by:

L.D. Wilson
Manager, Strategy Development
Corporate Power System Programming Division

C.F. Lee
Area Coordinator - Geotechnical

A.L. Punhani
Senior Mathematician
Reliability and Statistics Section

AHCC/LDW/ALP:djb

REFERENCES

1. Raven, K.D. Studies in Fracture Hydrology at Chalk River Nuclear Laboratories. Program for Geological Disposal of Radioactive Wastes. Contaminant Hydrogeology Section. Inland Waters Directorate. Environment Canada. 1977-79.

2. Martin, D., Piteau, D. and Herget, G. Pit Slope Manual Supplement 2-5-Structural Geology Case History; CANMET (Canada Centre for Mineral and Engineering Technology), CANMET Report 77-24. 29 p. December, 1977.

3. Reilly, J. MAPNET: A Mathematical Model Representing Fracture Network Structure. Ontario Hydro Research Division Report (to appear).

4. Tammemagi, H.Y., Kerford, P.S., Requeima and Temple, C.S. A Geological Reconnaissance Study of the Lac du Bonnet Batholith, Atomic Energy of Canada Limited. WNRE, Pinawa, Manitoba, R0E 1L0. July, 1979.

APPENDIX I

SPACING OF SIMULATED FRACTURES

The basic idea to generate sub-surface fractures using the vertical fracture frequency $Q_i(z)$ [(c) of Sec. 2.2] is to simulate these fractures with a proper spacing on the plane $z = -k\Delta$ so that the number of fractures intersecting the vertical line segment from $z = -k\Delta$ so that the number of fractures intersecting the $Q_i((k+1)\Delta) - Q_i(k\Delta)$. The mean spacing of the fractures, t_{ik}, on the vertical line is given by:

$$\bar{t}_{ik} = \frac{\Delta}{Q_i\ ((k+1)\Delta) - Q_i(k\Delta)} \tag{I.1}$$

Hence the mean horizontal spacing of fractures originating in or passing through the plane $z = -k\Delta$ is $\bar{t}_{ik}\left|\cos\bar{\omega}_i\right|$ (see Figure 13).

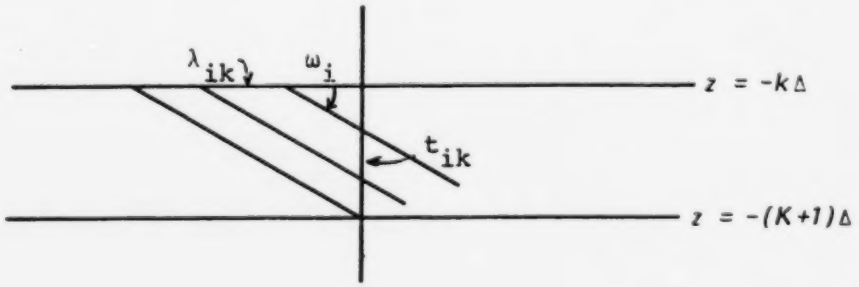


FIGURE 13

Taking into consideration of overlapping from the previous level defined by $(\bar{d}_i \sin\bar{\omega}_i)/\Delta$, $\bar{\lambda}_{ik}$, the adjusted mean spacing of the fractures generated on the plane $z = -k\Delta$, is

$$\bar{\lambda}_{ik} = \bar{t}_{ik}\left|\cot\bar{\omega}_i\right| \frac{\bar{d}_i \sin\bar{\omega}_i}{\Delta} \tag{I.2}$$

or

$$\bar{\lambda}_{ik} = \bar{t}_{ik} \left|\cos\bar{\omega}_i\right| \left(\frac{\bar{d}_i}{\Delta}\right) \tag{I.3}$$

Generate subsurface fractures with a mean spacing $\bar{\lambda}_{ik}$ in the same way as the surface fractures were generated. Generate a reference point on each horizontal line with mean spacing $\bar{s}_{ik}$ as:

$$\bar{s}_{ik} = \bar{\lambda}_{ik} \left(\frac{\bar{e}_i}{\Delta}\right) \tag{I.4}$$

Using (I.1) and (I.3)

$$\bar{s}_{ik} = \bar{t}_{ik} \left|\cos\bar{\omega}_i\right| \left(\frac{\bar{d}_i}{\Delta}\right) \cdot \left(\frac{\bar{e}_i}{\Delta}\right)$$

$$= \frac{\bar{d}_i \bar{e}_i \left|\cos\bar{\omega}_i\right|}{\Delta\left[Q_i((k+1)\Delta) - Q_i(k\Delta)\right]} \tag{I.5}$$

APPENDIX II

VERIFICATION OF THE SIMULATION MODEL

A simulation model is basically a mathematical analog of a natural phenomenon. In this study, it is intended to draw conclusions about the fracture network in rock surrounding a nuclear waste repository by studying the mathematical analog. It is, therefore, imperative that the model be as faithful a description of the real world as possible.

The yardstick for measuring the accuracy of the model is the comparison of the sampled occurrence of fractures with the observed occurrence. In the present case, the following propositions should hold:

Proposition 1: The mean spacing of the S_i fractures is $\bar{\lambda}_i$, $i = 1, 2, \ldots, I$.

Proof: To prove this proposition it is sufficient to show that the mean spacing of S_i fractures on each line perpendicular to the x-axis at j, $j = 1, 2 \ldots, \frac{D_1}{\Delta}$, is $\frac{\bar{\lambda}_i}{\sin\bar{\theta}_i}$.

Let L_o be an arbitrary North-South line segment at $j_o\Delta$ with length D_2 (see Figure 14). Furthermore, assume that $\alpha_j(y)$ is the characteristic function that y lies on a fracture whose location reference is $((j_o-j)\Delta, y, 0)$ and $\beta_j(y)$ is the characteristic function that y lies on a fracture having $e_i \sin\theta_i > j\Delta$.

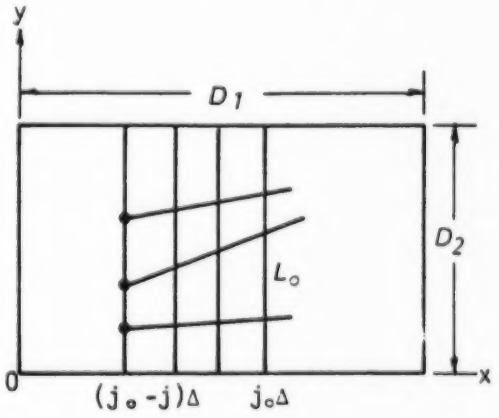


FIGURE 14

Then,

$$N'_j = \sum_{0 \le y \le D_2} \alpha_j(y)\, \beta_j(y) \qquad (II.1)$$

represents the number of fractures on the North-South line at (j_0-j) that intersect L_0. It should be noted here that the summation $\sum_{0 \le y \le D_2}$ is well defined because $\alpha_j(y) \cdot \beta_j(y) = 0$ for all

but countably many y's. Thus, $N' = \sum_{j=0}^{\infty} N'_j$ is the total number of fractures that intersect L_0.

Let,

$$p_j = P\{\beta_j(y) = 1\} = P\{\frac{e_i \sin\theta_i}{\Delta} > j\}. \qquad (II.2)$$

Now,

$$E(N'_j) = \sum_{m=0}^{\infty} E\{N'_j \mid \Sigma\alpha_j(y)=m\}\ P\{\Sigma\alpha_j(y) = m\} \qquad (II.3)$$

where,

$$E\{N'_j \mid \Sigma\alpha_j(y)=m\} = \sum_{n=0}^{m} n\ P\{N'_j = n \mid \Sigma\alpha_j(y) = m\} \qquad (II.4)$$

$$= \sum_{n=0}^{m} n \binom{m}{n} p_j^n (1-p_j)^{m-n}$$

$$= mp_j.$$

It follows that,

$$E(N'_j) = \sum_{m=0}^{\infty} E\{N'_j \mid \Sigma\alpha_j(y) = m\}\ P\{\Sigma\alpha_j(y) = m\} \qquad (II.5)$$

$$= \sum_{m=0}^{\infty} mp_j\ P\{\Sigma\alpha_j(y) = m\}$$

$$= p_j \sum_{m=0}^{\infty} m\ P\{\Sigma\alpha_j(y) = m\}$$

$$= p_j \varepsilon_i .$$

where ε_i is the mean of the number of fractures generated on line at $(j_0 = j)\Delta$ and this is given by

$$\varepsilon_i = \frac{D_2\Delta}{\bar{\lambda}_i \bar{e}_i} . \qquad (II.6)$$

Hence,

$$E(N') = \sum_{j=0}^{\infty} (N_j') = \sum_{j=0}^{\infty} P_j \, \varepsilon_i \qquad \text{(II.7)}$$

$$= \varepsilon_i \sum_{j=0}^{\infty} P_j = \varepsilon_i \sum_{j=0}^{\infty} P\{\frac{e_i \sin\theta_i}{\Delta} > j\}$$

$$= \varepsilon_i \frac{1}{\Delta} \sum_{j=0}^{\infty} \Delta j \; P\{j\Delta \leq e_i \sin\theta_i \leq (j+1)\Delta\}$$

$$= \frac{\varepsilon_i}{\Delta} E(e_i \sin\theta_i) \approx \frac{\varepsilon_i}{\Delta} \; \bar{e}_i \sin\bar{\theta}_i$$

$$= \frac{D_2 \Delta}{\bar{\lambda}_i \bar{e}_i} \quad \frac{\bar{e}_i \sin\bar{\theta}_i}{\Delta} = \frac{D_2 \sin\bar{\theta}_i}{\bar{\lambda}_i} \; . \qquad \text{(II.8)}$$

In other words, it is expected that $\frac{D_2 \sin\bar{\theta}_i}{\bar{\lambda}_i}$ fractures will intersect L_o. Hence, the mean spacing of fractures on L_o is

$$D_2/E(N') \text{ which is equal to } \frac{\bar{\lambda}_i}{\sin\bar{\theta}_i} \text{ (from II.8).} \qquad \text{(II.9)}$$

Proposition 2: The number of S_i fractures intersecting any vertical line segment with depth z is

$$Q_i^*(z) = Q_i(z) - \frac{\bar{d}_i \left|\cos\bar{\omega}_i\right|}{\bar{\lambda}_i} \; , \; i = 1, 2, \ldots, I \; . \qquad \text{(II.10)}$$

Proof: Consider an arbitrary point (x,y) on the surface. Let $z_k = -k\Delta$, $k = 1, 2, \ldots$. If a straight line is drawn at (x,y,z_k), for each $k = 1, 2, \ldots$, perpendicular to the mean strike angle $\bar{\theta}_i$, the mean spacing of the fractures generated at level k and intersecting this line is

$$\frac{\bar{d}_i \left|\cos\bar{\omega}_i\right|}{Q_i(z_{k+1}) - Q_i(z_k)} \; . \qquad \text{(II.11)}$$

Since the adjustment for previous levels overlap to k level is

$\frac{\bar{d}_i \sin\bar{\omega}_i}{\Delta}$, the actual spacing is

$$\frac{\Delta \left|\cot\bar{\omega}_i\right|}{Q_i(z_{k+1}) - Q_i(z_k)} \qquad \text{(II.12)}$$

In other words, there are $(Q_i(z_{k+1}) - Q_i(z_k))$ fractures expected to intersect the vertical line at (x, y) between k and (k+1) level. Similarly at the surface level, there are $(\bar{d}_i \cos\bar{\omega}_i)/\bar{\lambda}_i$ fractures intersecting the vertical line at (x,y). Hence the number of fracture intersecting the vertical line down to z_k is

$$\begin{aligned} Q_i^*(z_k) &= \sum_{j=0}^{k-1} \{Q_i(z_{j+1}) - Q_i(z_j)\} \qquad \text{(II.13)} \\ &= Q_i(z_k) - Q_i(z_0) \\ &= Q_i(z_k) - \frac{\bar{d}_i}{\bar{\lambda}_i} \left|\cos\bar{\omega}_i\right| . \end{aligned}$$

This completes the proof.

Proposition 3: The orientation of the surface fractures are distributed as shown in the stereoplot.

Proof: To prove this proposition, it is sufficient to show that the proportion of the fractures of the joint sets is the same as the stereoplot. This is quite easy to see, since

$$\frac{\text{Expected number of } S_i \text{ fractures generated}}{\text{Expected number of } S_j \text{ fractures generated}} = \frac{\bar{\lambda}_j}{\bar{\lambda}_i} , \qquad \text{(II.14)}$$

where

$$\frac{1}{\bar{\lambda}_i} = \frac{\text{Expected number of } S_i \text{ fractures on the survey line}}{\text{length of the survey line}} \qquad \text{(II.15)}$$

APPENDIX III

TRIAL APPLICATION OF THE FRACTURE GENERATION SIMULATION

1. INTRODUCTION

This appendix describes the assumptions made for a trial application of the fracture generation simulation model, undertaken to produce test data for Dr. W.G. Mathers' model of groundwater flow through discrete fractures.

2. OBJECTIVE

The purpose of the work is to gain experience with the model and its computer code. For that reason, no attempt is made to achieve a high degree of realism.

3. THE PRELIMINARY MODEL

For the purpose of testing the discrete fracture flow code, Dr. Mathers' required a fracture structure that consists of three joint sets including one set of horizontal fractures. Since there is no detailed fracture information available, in this preliminary model the necessary probability distributions and functions for the simulation were assigned somewhat arbitrarily. The distributions and functions used in this preliminary case are discussed below:

3.1 NUMBER OF SLOPING FRACTURE SETS

$I = 2$.

3.2 SIMULATION GRID SIZE

$\Delta = 1$ m.

3.3 SIZE OF STUDY ZONE

The study zone is a 10 m cube, ie,

$D_1 = D_2 = D_3 = 10$ m.

3.4 HORIZONTAL SPACING OF FRACTURES

Fracture spacing is assumed to follow a negative exponential distribution. Mean spacing data for the Lac du Bonnet batholith lie (see /4/) between 1 m and 3 m. Therefore, let

$$\Lambda_i(\lambda_i) = \int_0^{\lambda_i} \frac{1}{\lambda_i} \exp\left\{-\frac{s}{\lambda_i}\right\} ds \qquad \text{(III.1)}$$

where,

$$i = 1, 2;\ \bar{\lambda}_1 = 1.5 \text{ m and } \bar{\lambda}_2 = 2.5 \text{ m}.$$

3.5 STRIKE AND DIP ANGLES

The distributions Θ_i and Ω_i are basically empirical distributions observed from the stereoplots of fracture orientation. In an actual situation, their density functions are step functions (see Figure 15). Hence, choose Θ_i and Ω_i to be uniform distributions as follows:

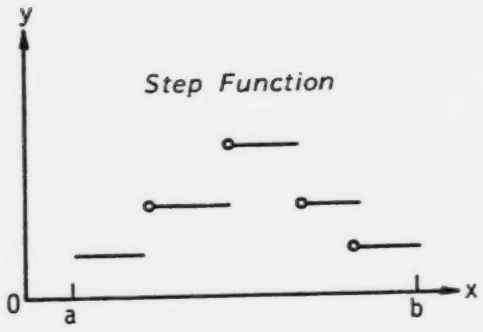


FIGURE 15

$$\Theta_i(\theta_i) = \frac{1}{2S_i}\ (\theta_i - \bar{\theta}_i + \delta_i), \qquad \text{(III.2)}$$

where,

$$\bar{\theta}_i - \delta_i \leq \theta_i \leq \bar{\theta}_i + \delta_i,\ \delta_i > 0 \text{ and } i = 1, 2\ .$$

$$\Omega_i(\omega_i) = \frac{1}{2\tau_i}\ (\omega_i - \bar{\omega}_i + \tau_i), \qquad \text{(III.3)}$$

where,

$$\bar{\omega}_i - \tau_i \leq \omega_i \leq \bar{\omega}_i + \tau_i,\ \tau_i > 0 \text{ and } i = 1, 2.$$

It is required that $\bar{\theta}_i > 0$ and $\bar{\omega}_i \neq \frac{\pi}{2}$. The values δ_i and τ_i are the ranges of variations of θ_i and ω_i, respectively. They must satisfy $0 \leq \bar{\theta}_i - \delta_i < \bar{\theta}_i + \delta_i \leq \pi$ and $0 \leq \bar{\omega}_i - \tau_i < \bar{\omega}_i + \tau_i \leq \pi$. The following values are selected:

$$\bar{\theta}_1 = \frac{\pi}{4}\ ,\ \delta_1 = \frac{\pi}{12} \qquad \text{(III.4)}$$

$$\bar{\theta}_2 = \frac{2\pi}{3}, \quad \delta_2 = \frac{7\pi}{36} \tag{III.5}$$

$$\bar{\omega}_1 = \frac{\pi}{6}, \quad \tau_1 = \frac{5\pi}{36} \tag{III.6}$$

$$\bar{\omega}_2 = \frac{11\pi}{18}, \quad \tau_2 = \frac{2\pi}{9} . \tag{III.7}$$

3.6 STRIKE LENGTH AND DIP LENGTH

There are indications in previous studies that strike lengths are negative exponentially distributed with mean $\bar{e}_i$ between 2 and 4 m. Therefore, let

$$E_i(e_i) = \int_0^{e_i} \frac{1}{\bar{e}_i} \exp\{-\frac{t}{\bar{e}_i}\}\, dt, \tag{III.8}$$

$$i = 1, 2 \text{ and } i = h.$$

Choose,

$$\bar{e}_1 = 2.5 \text{ m}$$

$$\bar{e}_2 = 3.5 \text{ m}$$

$$\bar{e}_h = 4 \text{ m}$$

where h represents the horizontal fracture. For the present exercise, define $\rho = 1.5$ for $i = 1, 2$ and $\rho = 1$ for $i = h$.

3.7 APERTURE

There is no information available on the aperture. Therefore, let a = 1 mm (constant aperture).

3.8 VERTICAL SPACING OF FRACTURES

No information on the vertical spacing of fractures is yet available to the author. Therefore, let the spacing be independent of depth. Let,

$$Q_i(z) = \alpha_i z,$$

for $i = 1, 2$ and $i = h$

and $\alpha_1 = \frac{1}{2}$ fractures/m,

$$\alpha_2 = \frac{1}{3} \text{ fractures/m}$$

and $\alpha_h = \frac{1}{4}$ fractures/m.

4. CONCLUSIONS

The above simulation model is constructed with minimum available fracture information. Many assumptions made here may not seem realistic, but are necessary for producing a simulated fracture structure. When more fracture data is available, the model can easily be modified and unrealistic assumptions can be dropped.

APPENDIX IV

MODIFICATION TO ACCOMMODATE THE OBSERVED LINEAMENTS

The following three cases of modifications are considered.

Case 1

When generating S_i fractures on the line $x = j\Delta$ and the plane $z = -k\Delta$, suppose that m fractures of joint set S_i, say $f_1, \ldots, f_m$, were identified as major lineaments at y_1, $y_2, \ldots, y_m$, as shown in Figure 16.

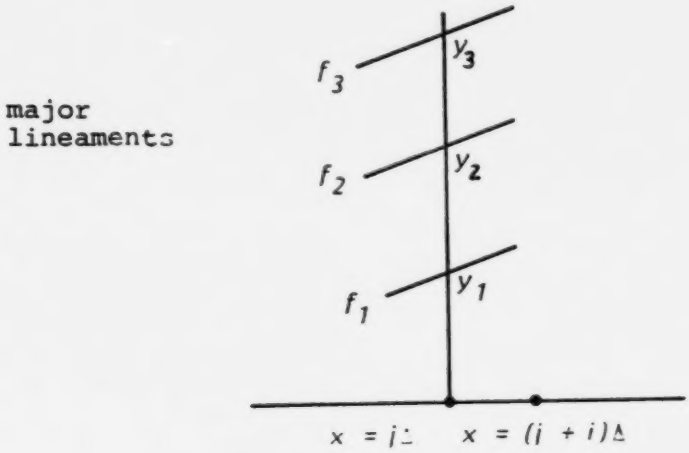


FIGURE 16

Define $y_0 = \min \{0, -\bar{e}_i \cos\bar{\theta}_i\}$ and $y_{m+1} = \max \{D_1, D_2 - \bar{e}_i \cos\bar{\theta}_i\}$. When generating S_i fractures on $x = j\Delta$, the inequality (2.1) of Section 2.3 needs to be modified. The basic idea is to generate S_i fractures only between y_{q-1} and y_q, for q = 1, 2, ..., m+1. Define n_1 by

$$\sum_{r=1}^{n_1} S_{ik}(j,r) < y_1 - y_0 < \sum_{r=1}^{n_1+1} S_{ik}(j,r)$$

And, in general, n_q is defined by

$$\sum_{r=1}^{n_q} S_{ik}(j,r) < y_q - y_{q-1} \leq \sum_{r=1}^{n_q+1} S_{ik}(j,r)$$

for all q = 1, 2, ..., m+1.

Let $n_{ik}(j) = n_1 + \ldots + n_{m+1}$. For the location reference points, they are defined by

$$P(x,y,z) = P(j\Delta, \sum_{r=n_{R'}+1}^{R} S_{ik}(j,r) + y_{R'}, -y_0, -k\Delta).$$

where R' is such that $n_1 + \ldots + n_{R'} < R \leq n_{R'+1}$ (see Figure 17).

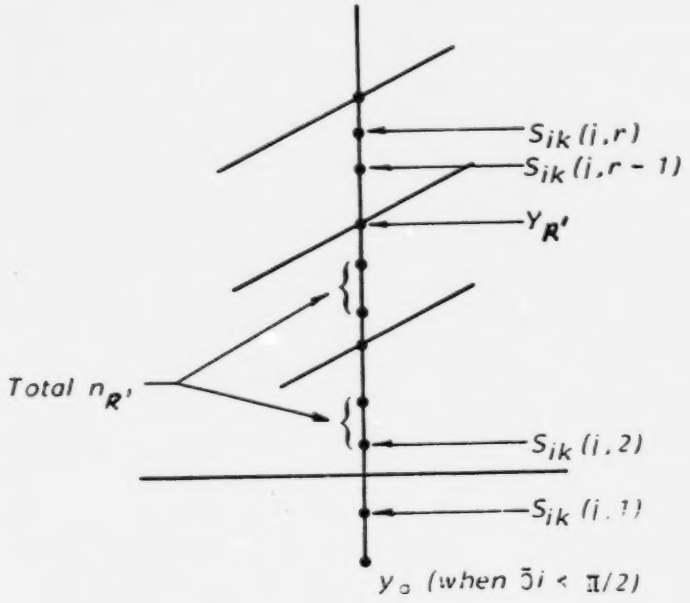


FIGURE 17

Case 2 When generating fractures for joint set $S_{i'}$ on the line x = jΔ and the plane z = -kΔ and major lineaments were identified to intersect this line only on the joint set S_i ($i \neq i'$), no modification is then needed.

Case 3 Suppose m horizontal fractures are observed in a borehole at the points (x',y') having depth z_q with apertures a_q, q = 1, 2 ..., m. First select m random numbers $e_1, \ldots, e_m$ from E_h. For each e_q, q = 1, 2, ..., m, select a pair of random numbers (x_q'', y_q'') from uniform distribution $U(-e_q, 0)$ $U(-e_q, 0)$. Put $x_q = x' + x_q''$ and $y_q = y' + y_q''$ (Figure 18).

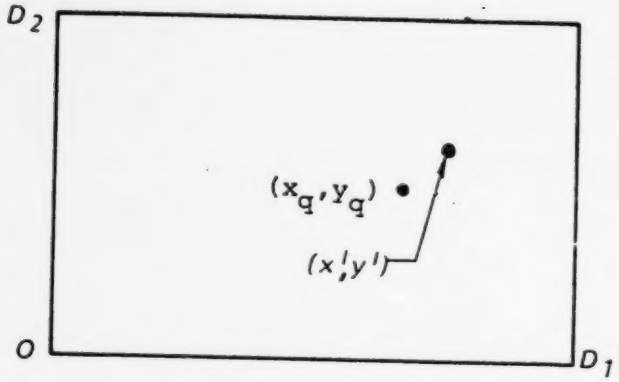


FIGURE 18

Define these fractures as

$$(x_q,\ y_q,\ z_q,\ 0,\ \pi,\ e_q,\ e_q,\ a_q),$$

for q = 1, 2, ..., m.

Let,

$$B' = \sum_{q=1}^{m} (\min\{x_q+e_q, D_1\}-\max\{0,x_q\})\ (\min\{y_q+e_q+D_2\}-\max\{0,y_q\})$$

Let AR be the area as defined in Figure 19. Then

$$AR = \{(x,y):\ \ |x-x'| < \frac{\bar{e}_h}{2} \text{ and } |y-y'| < \frac{\bar{e}_h}{2}\ \}$$

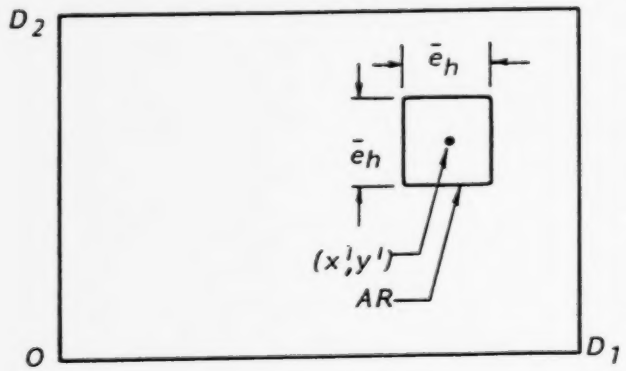


FIGURE 19

Then select a set of random numbers $(x_n,\ y_n,\ z_n)$ from the uniform distribution

$$U(-\bar{e}_h, D_1)\ U(-\bar{e}_h, D_2)\ .$$

If (x_n, y_n) lies in AR, discard (x_n, y_n) and select another set until (x_n, y_n) lies outside AR. Then select z_n from Q_h', e_n from E_h and a_n from A_h. Assign the fracture as

$$(x_n, y_n, z_n, 0, \pi, e_n, e_n, a_n)$$

Define ε_n as in Step 4 of Section 2.3. Then, the sampling is terminated at n_h when

$$\sum_{n=1}^{n_h} \varepsilon_n \leq BD_1D_2 - B' \leq \sum_{n=1}^{n_h+1} \varepsilon_n$$